RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, MARCH 2021
THIRD YEAR [BATCH 2018-21]
PHYSICS (Honours)
Paper : V [Gr. A]

Date : 13/03/2021
Time : 11 am - 1 pm
Full Marks : 50

Answer **any five** questions: [5 × 10]

1. a) State the postulate of equal a priori probability in equilibrium statistical mechanics. Determine and plot the phase-space trajectory of a particle freely falling under gravity. [1+2]

   b) Consider a system of three spin ½ particles each having magnetic moment μ in an external magnetic field H (along say, z-direction). Each particle may orient along or opposite to the external field.

   i) List all possible microstates and macrostates.

   ii) If the total energy of the system is known to be +μH, what are the possible states and what is the probability that the spin of the first one will point up? [2+2]

   c) Consider a one level system having energy $\epsilon = -k_B T \ln(V/V_0)$ where V_0 is a constant. Write down the partition function for this system and calculate the average pressure as a function of volume and temperature. [3]

2. a) In what way does the Fermi-Dirac distribution differ from the Maxwell-Boltzmann distribution. [2]

   b) Name the statistics (BE or FD) obeyed by each of the following particles: [3]

   proton, muon, phonon, α-particle, neutrino

   c) In the domain of quantum statistics, derive the validity criterion of classical (M-B) approximation.

   At the centre of the sun temperature $T \sim 10^7$ K, and concentration of electrons $n \sim 10^{32}\ m^{-3}$. Would it be valid to treat those electrons as a classical ideal gas?

   [Given: rest mass of electron = 9.11×10^{-31} Kg, Planck's constant (h) = 6.62×10^{-34} Js,

   Boltzmann constant (k_B) = $1.38\times10^{-23}\ JK^{-1}$.] [3+2]

3. a) Distinguish between ordinary vapour-liquid condensation and Bose-Einstein condensation. Physically explain the phenomena of B-E condensation. [2+2]

   b) Chemical potential of boson should always be negative – justify this statement. Plot the approximate variation of the chemical potential with temperature. Find out an expression of B-E condensation temperature. [2+4]

4. a) Prove that for a system at T > 0 K obeying F-D statistics, the probability that a level lying $\Delta\epsilon$ below the Fermi level is unoccupied is the same as the probability of occupation of a level lying $\Delta\epsilon$ above the Fermi level. [2]

b) Consider a non-interacting Fermi gas confined in a volume V at temperature T=0K. Derive expressions of (i) Fermi energy (E_F), (ii) Total energy of the system and (iii) the degeneracy pressure of the Fermi gas. [2+2+2]

c) Estimate the Fermi energy of silver atom having atomic weight 107.87 and density 10.5 gm/cc. Assume that each silver atom denotes one conduction electron [2]

5. a) Consider a photon gas enclosed in a volume V and in equilibrium at temperature T. With the help of B-E statistics find out the energy density of the photon gas as a function of wavelength (λ). Depict the nature of variation. [5+1]

b) Using above expression establish Wien's displacement law.

At a given temperature, λ_{max} = 640 nm, for a cavity. What will be the value of λ_{max} , if the temperature of the cavity walls is increased so that the rate of emission of spectral radiation is doubled? [2+2]

6. a) Set up Langevin's equation for one dimensional Brownian motion. Solve the equation with the help of reasonable approximations [Justify all those approximations]. Find the mean square displacement of the Brownian particle. What is the importance of the final result? [7]

b) Show that the variance of energy according to Maxwell-Boltzmann energy distribution law is $\frac{3}{2}\left(\frac{k_B}{T}\right)^2$. [3]

7. a) Write down the expression of canonical partition function Z. Find the internal energy and entropy in terms of Z. [1+3]

b) Write down the grand partition function. Derive the Fermi-Dirac and Bose-Einstein distribution from the grand partition function. [1+5]

8. A system with N non interacting particles has two energy levels at energies 0 and ϵ having degeneracy g_0 and g_1 respectively.

a) Write down the partition function. [1]

b) Calculate the average energy and entropy of the system. [3]

c) Calculate the specific heat of the system. Figure out the high temperature behaviour and the low temperature behaviour of specific heat. [4+2]

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